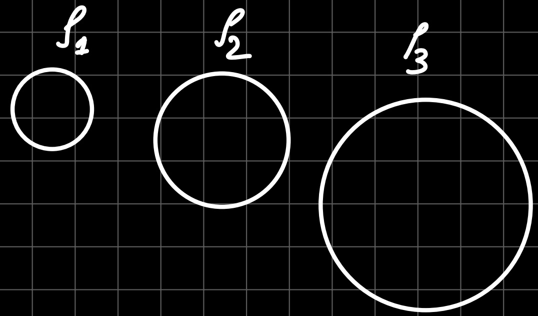
Fermi liquid theory

* m^* determination $\Rightarrow$ TG supplies a calculation

$$\frac{m^*}{m} = 1 + \frac{1}{3} F^s_{l=1}$$

Zero sound



$\rightarrow \rho^{1/3} \sim k_F$

$\rightarrow$ can be thought as a fluctuations of fermi surface

$$\underset{\substack{\uparrow \\ \text{lifetime in QP}}}{\omega\tau} \ggg 1$$

* We try to capture this using semi-classical boltzmann eqn.

$$n_p(r,t) = n_p^0 + \underbrace{\delta n_p(\vec{q},\omega)}_{\text{assume one particular mode vibrates}} e^{i(\vec{q}\cdot\vec{r} - \omega t)}$$

boltzmann eqn:-

$$\frac{\partial g}{\partial t} + \frac{\partial g}{\partial r}\cdot\frac{\partial \vec{r}}{\partial t} + \frac{\partial g}{\partial \vec{k}}\cdot\frac{\partial \vec{k}}{\partial t} = \left.\frac{\partial g}{\partial t}\right|_{\text{collision term}}$$

$$= -\int dk' \Big\{ \underbrace{W_{k\to k'}}_{\substack{\text{amplitude of} \\ \text{going from} \\ k \text{ to } k'}} g_k \{1 - g_{k'}\} - W_{\vec{k}'\to\vec{k}} (1 - g(\vec{k})) g_{\vec{k}'} \Big\}$$

in $\omega\tau \ggg 1$ there are no collisions

now we apply rel. time approximation $\approx -\dfrac{\{g(k)-g^0(k)\}}{\tau}$

$(g \approx n)$

<u>Collision free limit</u>:-

$$\frac{\partial}{\partial t} n_p + \frac{\partial n_p}{\partial \vec{r}} \cdot \frac{\partial \vec{r}}{\partial t} + \frac{\partial n_p}{\partial \vec{p}} \cdot \frac{\partial \vec{p}}{\partial t} = 0$$

$$\left.\frac{d\vec{r}}{dt} = \vec{v} = \frac{\partial \mathcal{E}_p}{\partial p} \qquad \frac{\partial \vec{p}}{\partial t} = \frac{-\partial \mathcal{E}_p}{\partial r} \right\} \text{from hamilton's eq}^n$$

FLT:

$$\xi_p = \xi_p^0 + \sum_{p'} f_{pp'} \, \delta n_{p'}$$

$$n_p = \underbrace{n_p^0}_{\text{@ low temp}} + \delta n_p(\vec{r}, t)$$

$$\Theta\left(E_F - \frac{p^2}{2m^*}\right)$$

$$\Rightarrow \frac{\partial \delta n_p}{\partial t} + \frac{\partial \delta n_p}{\partial p} \cdot \frac{\partial \mathcal{E}_p}{\partial p} - \frac{\partial n_p^0}{\partial \vec{p}} \sum_{p'} f_{pp'} \frac{\partial \delta n_p}{\partial \vec{r}} = 0$$

now $\delta n_p(\vec{r}, t) = \delta n_p \exp(i\vec{q}\cdot\vec{r} - i\omega t)$

$$(\vec{q}\cdot\vec{v}_p - \omega)\,\delta n_p - \vec{q}\cdot\vec{v}_p \frac{\partial n_0}{\partial \mathcal{E}_p} \sum f_{pp'} \delta n_p = 0$$

$$\vec{q} = q\hat{z} \qquad \therefore \vec{q}\cdot\vec{v}_p = \frac{qP_F}{m^*}\cos\theta$$

$$\delta n_p = \delta(\xi_p - E_F)\, v_F u_p \qquad \frac{\partial n_0}{\partial \xi_p} = \delta(\xi_p - E_F)\, v_F u_p$$

$$(q v_F \cos\theta - \omega)\, u(\theta) - q v_F \cos\theta \int \frac{d\Omega}{4\pi}\, f(\theta-\theta')\, u(\theta') = 0$$

$$\lambda = \frac{\omega}{q v_F}$$

$\times D(E_F)$

$$(\cos\theta - \lambda)\, u(\theta) - \frac{\cos\theta}{2} \int f(\theta-\theta')\, u(\theta') \sin\theta'\, d\theta' = 0$$

Let's assume that only F_0 is non-zero.

$$\boxed{u(\theta) = \frac{F_0\, x}{2(\cos\theta - \lambda)}} \qquad x = \int u(\theta') \sin\theta'\, d\theta'$$

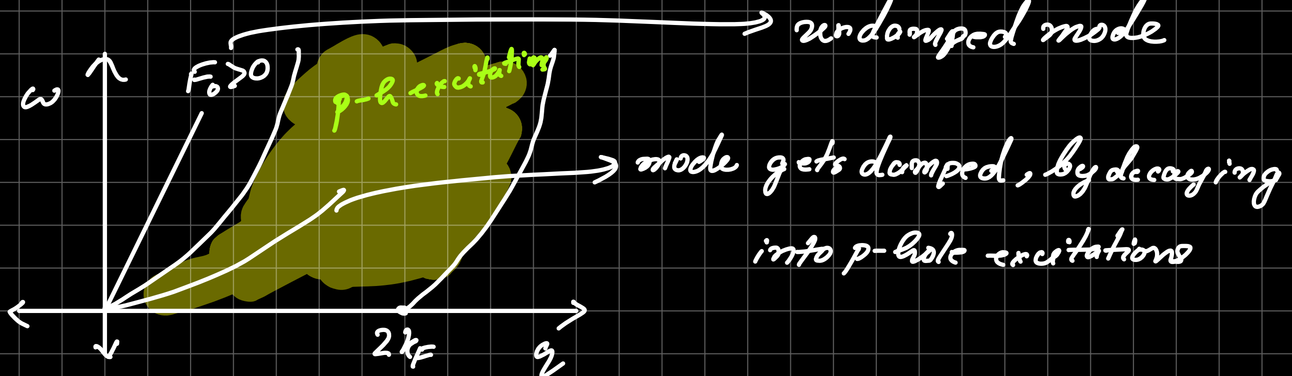
$$\Rightarrow \quad 1 = \frac{F_0}{2} \int \frac{\sin\theta\, d\theta}{(\cos\theta - \lambda)} \quad \Rightarrow \quad \frac{2}{F} = -\ln|\cos\theta - \lambda| \Big|_0^{\pi}$$

$$= -\ln\left(\frac{-1-\lambda}{1-\lambda}\right)$$

$$\Rightarrow \quad \frac{-2}{F} = \ln\left(\frac{\lambda-1}{\lambda+1}\right)$$

$$e^{-\frac{2}{F}} = 1 - \frac{2}{\lambda+1} \quad \Rightarrow \quad \frac{2}{\lambda+1} = 1 - e^{-\frac{2}{F_0}} \quad \Rightarrow \quad \lambda + 1 = \frac{2}{1 - e^{-\frac{2}{F_0}}}$$

$$\Rightarrow \quad \boxed{\lambda = \frac{2}{1 - e^{-\frac{2}{F_0}}} - 1}$$



Boltzman eqn only for small k_F & small ω

$\Rightarrow$ Chetan Nayak's notes features a microscopic derivation.

Plasmon:- gapped in 3d, gapless in 2D

* Dirac particles:- int. is irrel.

$\hookrightarrow$ Coulomb int. is long ranged (not a good screening)

power counting

$$\int d^2x\, d\tau\, \bar{\psi}\gamma\psi + \int(\bar{\psi}\psi)^2 d^2x\, d\tau$$

* marginal FL

Luttinger liquid, GHIDs, SMs

* Sean Hartnoll et al

FL → mott insulator

"Bad metals"

↓

high temp processes

* "ersatz fermi liquids"

"anomalies"

"elsewhere"

* Gapped phases have more progress

* Qns abt FS are diff to see

Large FS (must have large symmetry group)

* Hubbard model at large U
↳ mott phase

mean field → ∞ diagrams

weak interactions

different things

*Umklapp scattering

H_v to pickup these terms